Name Key
PreCalculus

Date: _____
Matrices Review

1. Solve the following system of equations using Cramer's Rule.

$$2x-3y+z=-7$$
$$4x-2y-z=-9$$
$$x+y+2z=3$$

$\begin{bmatrix} 2 & -3 & 1 \\ 4 & -2 & -1 \\ 1 & 1 & 2 \end{bmatrix}$ det = 27

x: $\begin{bmatrix} -7 & -3 & 1 \\ -9 & -2 & -1 \\ 3 & 1 & 2 \end{bmatrix}$ det = -27, $x = \frac{-27}{27} = \boxed{-1}$

y: $\begin{bmatrix} 2 & -7 & 1 \\ 4 & -9 & -1 \\ 1 & 3 & 2 \end{bmatrix}$ det = 54, $y = \frac{54}{27}$, $y = \boxed{2}$

z: $\begin{bmatrix} 2 & -3 & -7 \\ 4 & -2 & -9 \\ 1 & 1 & 3 \end{bmatrix}$ det = 27, $z = \frac{27}{27}$, $\boxed{z = 1}$

2. Solve the following system of equations using row reduction.

$$2x-3y+z=11$$
$$4x-2y-z=5$$
$$x+y+2z=5$$

$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ $x = 1$, $y = -2$, $z = 3$

3. Solve the following system of equations using an inverse matrix.

$$4x-y=10$$
$$2x+2y=30$$

$\begin{bmatrix} 4 & -1 \\ 2 & 2 \end{bmatrix}$ (A) $\begin{bmatrix} x \\ y \end{bmatrix}$ (X) $=$ $\begin{bmatrix} 10 \\ 30 \end{bmatrix}$ (B)

$\boxed{x = 5,\ y = 10}$

$X = A^{-1} \cdot B$

4. Using matrices, find an equation with integer coefficients that passes through $(5,6)$ and $(-1,-6)$.

$6x - 6x + 5y$

$6x - y - 30 + (+6 + 6x - 5y)$

$12x - 6y - 24 = 0$

$6x - y - 30$

5. Using matrices, find the area of the triangle whose vertices are $(5,6)$, $(-1,-6)$, and $(1,10)$.

$-6 - 30 - 6 = -38$

$-30 + 6 - 10 = -34$

$-34 - 38 = -72 \times \frac{1}{2} = -36$

$\boxed{\text{Area} = 36 \text{ units}^2}$

6. Are the points $(5,6)$, $(-1,-6)$, and $(2,1)$ collinear? Justify with matrices.

$-12 + 5 - 6 = -13$

$-30 + 12 - 1 = -19$

$-19 - -13 = -6$ $\boxed{\text{No}}$

7. Find the determinant of $E = \begin{bmatrix} g & h & i \\ a & b & c \\ d & e & f \end{bmatrix}$

$dbi + ecg + fah$

$gbf + hcd + iae$

$\boxed{(gbf + hcd + iae) - (dbi + ecg + fah)}$

Questions 8 – 17 refer to the following matrices.

$$A=\begin{bmatrix}4 & 5\\ -1 & 3\end{bmatrix} \quad B=\begin{bmatrix}0 & -2\\ 1 & -3\end{bmatrix} \quad C=\begin{bmatrix}2\\ -3\end{bmatrix} \quad D=\begin{bmatrix}1 & 2\end{bmatrix}$$

8. $A+3B$. $\begin{bmatrix}4 & -1\\ 2 & -6\end{bmatrix}$

9. $C+D$ Undefined

10. $A-C$ Undefined

11. A^{-1} $\begin{bmatrix}\frac{3}{17} & -\frac{5}{17}\\ \frac{1}{17} & \frac{4}{17}\end{bmatrix}$

12. $|A| = 17$

13. AB^{-1} $\begin{bmatrix}-\frac{17}{2} & 4\\ 0 & -1\end{bmatrix}$

14. BC $\begin{bmatrix}6\\ 11\end{bmatrix}$

15. CB Undefined

16. DA $\begin{bmatrix}2 & 11\end{bmatrix}$

17. BD Undefined

18. Write the system of equations whose representation is $\begin{bmatrix}2 & -3 & 1\\ 4 & -2 & -1\\ 1 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 5\\ 5\end{bmatrix}$.

$2x - 3y + z = 1$
$4x - 2y - z = 5$
$x + y + 2z = 5$

19. Solve the following system of equations using Cramer's Rule.

$$-x+3y-2z=0$$
$$2x-y+z=-1$$
$$x+y+-z=-2$$

$\begin{bmatrix}-1 & 3 & -2\\ 2 & -1 & 1\\ 1 & 1 & -1\end{bmatrix}$ det: 3

x: $\begin{bmatrix}0 & 3 & -2\\ -1 & -1 & 1\\ -2 & 1 & -1\end{bmatrix}$ det: -3 $x = \frac{-3}{3} = \boxed{-1}$

y: $\begin{bmatrix}-1 & 0 & -2\\ 2 & -1 & 1\\ 1 & -2 & -1\end{bmatrix}$ det: 3 $y = \frac{3}{3} = \boxed{1}$

z: $\begin{bmatrix}-1 & 3 & 0\\ 2 & -1 & -1\\ 1 & 1 & -2\end{bmatrix}$ det: 6 $z = \frac{6}{3} = \boxed{2}$

20. Solve the following system of equations using row reduction.

$$-x+3y-2z=15$$
$$2x-y+z=-3$$
$$x+y+-z=9$$

$\begin{bmatrix}1 & 0 & 0 & 2\\ 0 & 1 & 0 & 3\\ 0 & 0 & 1 & -4\end{bmatrix}$ $x=2$ $y=3$ $z=-4$

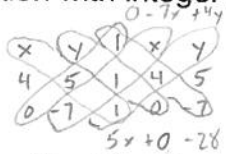
21. Solve the following system of equations using an inverse matrix.

$$4y-x=-14$$
$$3x-2y=12$$

$\begin{bmatrix}-1 & 4\\ 3 & -2\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-14\\ 12\end{bmatrix}$ $\begin{bmatrix}2\\ -3\end{bmatrix}$ $x=2$ $y=-3$

$x = A^{-1}\cdot B =$ $\begin{bmatrix}-8\\ -18\end{bmatrix}$ $x=-8$ $y=-18$

22. Using matrices, find an equation with integer coefficients that passes through $(4,5)$ and $(0,-7)$.



$(5x - 28) - (-7x + 4y)$

$12x - 4y - 28 = 0$

23. Using matrices, find the area of the triangle whose vertices are $(4,5),(0,-7)$, and $(1,10)$.

$\det = -56$

Area = $28\sqrt{4}$

24. Are the points $(5,6),(-1,-6)$, and $(2,0)$ collinear? Justify with matrices.

$\det = 0$ yes

25. Find the determinant of $E = \begin{bmatrix} d & e & f \\ a & b & c \\ g & h & i \end{bmatrix}$

$(dbi + ecg + fah) - (fbg + dch + eai)$

Questions 26 – 35 refer to the following matrices.

$$A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} -1 & -2 \\ -5 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 3 \end{bmatrix}$$

26. $A + 3B$. $\begin{bmatrix} -3 & -4 \\ -16 & 18 \end{bmatrix}$

27. $C + D$ Undefined

28. $A - C$ Undefined

29. A^{-1} $\begin{bmatrix} \frac{3}{2} & -1 \\ \frac{1}{2} & 0 \end{bmatrix}$

30. $|A| = 2$

31. AB^{-1} $\begin{bmatrix} -\frac{2}{3} & \frac{2}{15} \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$

32. BC $\begin{bmatrix} 2 \\ -20 \end{bmatrix}$

33. CB undefined

34. DA $\begin{bmatrix} -3 & 13 \end{bmatrix}$

35. BD undefined